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Brief communication

Wave propagation in physiological collapsible tubes and a proposal for a Shapiro number

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Abstract

In this Brief Communication, a few introductory comments are made first, regarding wave propagation in collapsible tubes and flow limitation, as well as what is commonly referred to as the "speed index," namely the ratio of the flow velocity in the collapsible tube divided by the wave-propagation velocity. It is then proposed that it is fitting for this speed index to be henceforth named as the *Shapiro number.*

Keywords: Collapsible tube; Wave propagation; Haemodynamics; Shapiro number

1. Introduction

The recent publication of a paper by Marzo et al. (2005) gives me a long-sought opportunity to propose the coining of a "Shapiro number".

In that paper, among other things, the wave velocity in fluid-filled collapsible tubes is discussed,

$$c = \left(\frac{A}{\rho}\frac{\mathrm{d}p}{\mathrm{d}A}\right)^{1/2}, \tag{1}$$

where A is the mean cross-sectional area of the tube, p is the transmural pressure in the tube (the inner minus the outer pressure), and ρ the fluid density. In the physiological milieu this is known as the Young equation, in honour of the pioneering work on blood pressure pulses at the beginning of the 19th century by Young (1808). Perhaps the first derivation of this equation was by Weber (1866) in conjunction with work motivated by the problem of propagation of arterial pulse waves.[1] The same equation crops up in the study of wave propagation in veins, pulmonary passages and the urinary tract [see reviews by Skalak (1966), Pedley (1977), Kamm (1987), Skalak et al. (1989), Kamm and Pedley (1989), Grotberg (1994), Bertram (1995); see also Fung (1966), Pedley (1980), and Carpenter and Pedley (2003)]. Also, it crops up in wave propagation in non-physiological pliable tubular systems; for example in collapsible sausage-like containers used for transporting lighter than sea-water fluid cargo (Hawthorne, 1961; Païdoussis, 1965, 2003).

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[1]In fact, there was *a series* of substantial contributions leading to Eq. (1): by Euler in 1775 [published posthumously in Euler (1862)], Young (1808), Résal (1876), Weber and Weber (1825), Weber (1850, 1866); experimental confirmation was provided, also long ago, by Moens (1878) and Korteweg (1878).

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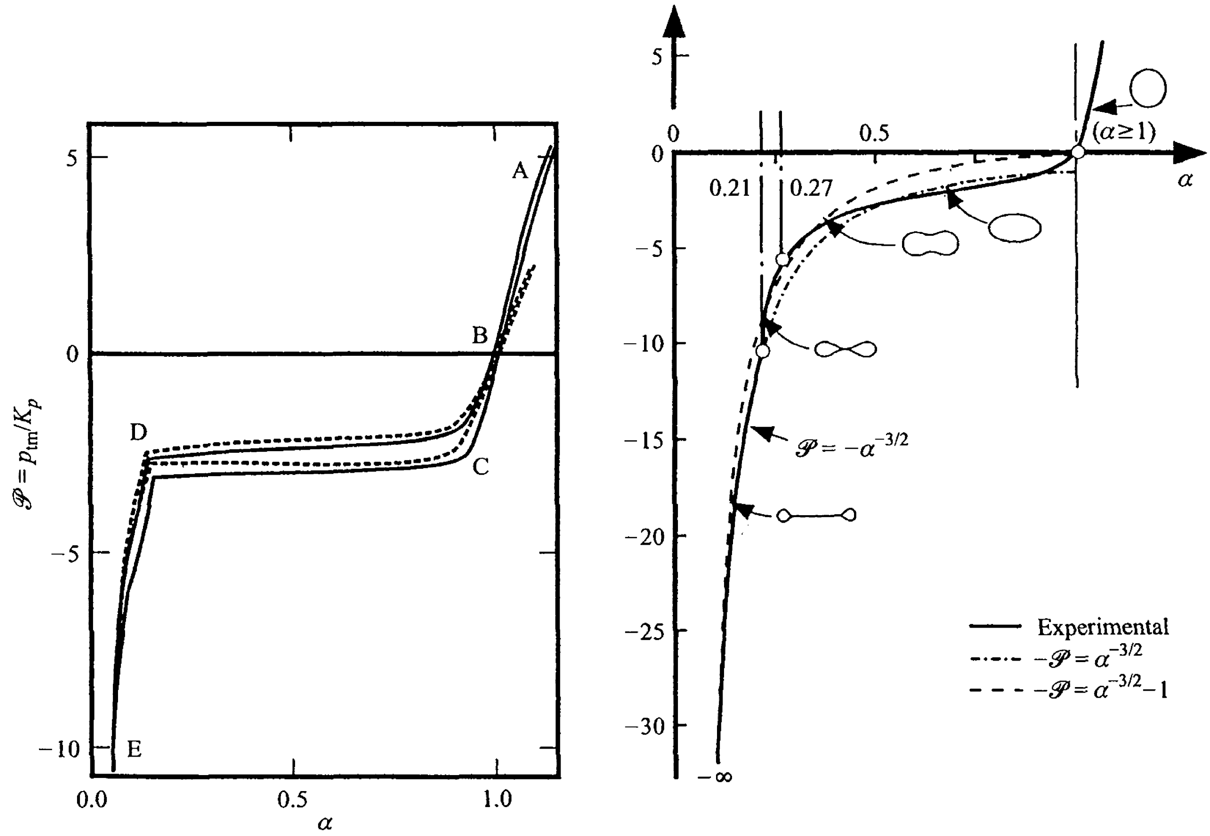

Fig. 1. The relation of $\mathscr{P} = p_{tm}/K_p$ and α, where p_{tm} is the transmural pressure, K_p a parameter proportional to the bending stiffness of the tube, and α is the 'fractional filling' equal to the actual cross-sectional area divided by the circular cross-sectional area of a tube with the same perimeter, $A/[(\pi/4)D^2]$. (a) For relatively thick elastomer tubes of two different thicknesses: —, thinner; – – –, thicker (Bertram, 1995); (b) for much thinner tubes, also showing typical cross-sectional shapes and analytical relationships (Shapiro, 1977a,b). It should be noted that Bertram's curves in this figure are based on observed data; Shapiro's curves, on the other hand, are idealizations—successful ones, as it turned out—not based on experimental data (unavailable at the time).

The principal characteristic of pliable, collapsible tubes is that, as the pressure therein changes, so does the cross-sectional area; see Fig. 1.

Similarity of Eq. (1) to that for propagation of waves in a compressible gas is evident once it is realized that such waves involve variations in density rather than the cross-sectional area; so, if A is replaced by ρ, this results in $c = (\mathrm{d}p/\mathrm{d}\rho)^{1/2}$.

Ascher Shapiro, who, as we all know, started in gas dynamics [*vide* his classic two-volume text (Shapiro, 1953, 1954)], later switched his attention to physiological problems. His two papers on the dynamics of collapsible tubes (Shapiro, 1977a,b) are landmarks in the study of fluid-structure interactions in pliable physiological tubular passages: veins, arteries, pulmonary passages, the urethra and the ureter. As Bertram (1999) quite rightly pointed out, these two papers, by their quality, pertinence and breadth, "have resisted all attempts in what ISI[2] would call obliteration by incorporation".

An interesting phenomenon arising in collapsible tubes with internal flow is that of *flow limitation*. As the upstream pressure p_1 is increased relative to the downstream one p_2, thus as $p_{1-2} = p_1 - p_2$ is increased, there exists a limit in the flow-rate Q; if p_{1-2} is increased further, no further increase in Q is possible. This, obviously, is very counter-intuitive from the point of view of rigid-pipe hydraulics!

As seen in Fig. 2, obtained from experimental measurements by Bertram (1995), for a constant transmural pressure at the upstream end of the tube, denoted by p_{1-e} (e standing for external), and varying p_{2-e}, as p_{1-2} increases there is a limit for Q, a maximum value; three such cases are shown in the figure for three different values of p_{1-e}. In fact, the flow

[2]Institute for Scientific Information.

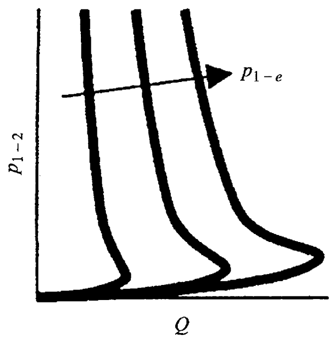

Fig. 2. The relationship between the upstream-to-downstream pressure difference p_{1-2} in a collapsible tube conveying fluid; Q is the volumetric flow rate, and p_{1-e} is the transmural pressure at the upstream end which has a different fixed value for each of the three curves shown (Bertram, 1995). The curves in this case are artistic approximations of what has been typically observed.

rate decreases somewhat with increasing p_{1-2} beyond that yielding the maximum Q. This corresponds to the so-called "negative effort dependency" in maximal flow expiration (Grotberg, 1994). In an appealing analogy, flow limitation is likened to flow over a waterfall, where the flow-rate is independent of the level of the downstream reservoir.

One of the mechanisms associated with flow limitation in collapsible tubes is the so-called *wave-speed mechanism* (Shapiro, 1977b; Wilson et al., 1986; Païdoussis, 2003), with strong analogy to gas dynamics. One may calculate

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \frac{A}{\rho U}\left(1 - \frac{U^2}{c^2}\right), \tag{2}$$

where Q is the volumetric flow-rate in the collapsible tube, p the transmural pressure, A the cross-sectional area, U the flow velocity corresponding to any particular A (both A and U vary along the tube), and c the wave speed as in Eq. (1). Shapiro calls U/c the "speed index" S, and so Eq. (2) is re-written as

$$\frac{\mathrm{d}Q}{\mathrm{d}p} = \frac{A}{\rho U}\left(1 - S^2\right). \tag{3}$$

It is obvious that as $S \to 1$ there is a discontinuity associated with a maximum flow rate in the collapsible tube.[3] This is also associated with so-called elastic jumps (Kececioglu et al., 1981; McClurken et al., 1981; Wilson et al., 1986; Bertram, 1995). Thus, in all this, the analogy to choked flow and shock waves in gas dynamics is quite strong.

Before closing this section, it is perhaps useful to mention some of the limitations of what has been presented. First, it should be stressed that Eq. (1) and everything else that has been discussed in the foregoing, strictly applies to one-dimensional wave propagation and one-dimensional flow. Then, it should be said that many interesting aspects of the dynamics have been left out. For example, in conjunction with Fig. 1, the discontinuities in the tube-law curves are not incidental [see Bonis and Ribreau (1981) and Ribreau et al. (1993)]. Of course, these peculiarities in the tube law result in a nonmonotonic variation in the wave speed as the transmural pressure is varied [elucidated by Brower and Scholten (1975) and Bonis and Ribreau (1981)]. Indeed, many other aspects of this fascinating subject have been left out, the main objective being to highlight, in the briefest possible manner, some of Shapiro's contributions.

2. A proposal for a Shapiro number

Clearly, from the foregoing, the speed index S plays the same role in collapsible tube dynamics as the Mach number in gas dynamics and the Froude number in hydraulics.

While writing the appropriate chapter in a book, having read the pertinent literature, some afresh and some for the first time, the seminal contributions to the field by Shapiro were very clear in my mind, and so I wrote in a footnote "Perhaps the time is now ripe—in fact, it has been so for a long time—to call S the *Shapiro number*" (Païdoussis, 2003, p. 756).

[3]The concept of a "sonic cross-section" in flow through collapsible tubes actually first appeared in studies earlier than Shapiro's; e.g., in those of Griffiths (1971, 1975) and Comolet (1976).

It is this proposal that is now repeated in this Communication in a more formal way, and it will hopefully reach a much wider readership among those working in physiological fluid-structure interactions than did the book.

3. Conclusion

Briefly, what is proposed in this Brief Communication is to rename the speed index S as the Shapiro number, in honour of Shapiro's very important contributions in the field of physiological fluid-structure interactions.

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